Using Simpson's Rule for the normal distribution

This problem uses Simpson's rule to approximate a definite integral important in probability.

In our probability unit, we learned that when given a probability density function $f(x)$, we may compute the probability P that an event x is between a and b by calculating the definite integral:

$$P(a \leq x \leq b) = \int_a^b f(x)\, dx.$$

Here we're assuming that a probability density function $f(x)$ has the property that

$$\int_{-\infty}^{\infty} f(x)\, dx = 1.$$

In the next session, we will show that $f(x) = \frac{1}{\sqrt{\pi}} e^{-x^2}$ is a probability density function with this property. For now, we assume this property.

Question: Suppose the probability density function for American male height is roughly (in inches x)

$$h(x) = \frac{1}{2.8\sqrt{2\pi}} e^{-(x-69)^2/5.6}.$$

- Use Simpson's rule to estimate the probability that an American male is between 5 and 6 feet tall.
- Use Simpson's rule to estimate the probability that an American male is over 8 feet tall.

24/9/25

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$P(5 \le x \le 6)$

$= \int_{60}^{72} \frac{1}{2.8\sqrt{2\pi}} e^{-(x-69)^2/5.6}$

$n=2,\ \Delta x = \frac{12}{2} = 6$

5ft = 60 inches

6ft = 72 inches

$\approx \frac{\Delta x}{3}(y_0 + 4y_1 + y_2)$

$= \frac{6}{3}(7.447\times10^{-8} + 4(0.02856) + 0.02856)$

$= 0.2856$

$n=4,\ \Delta x = 3 \Rightarrow P(5 \le x \le 6) = 0.6565$

$n=6,\ \Delta x = 2 \Rightarrow P(5 \le x \le 6) = 0.5742$

8 ft = 96 inches

8ft 4 inches = 100 inches

$P(x > 8)$

$\approx \int_{96}^{100} h(x)\,dx$

$n=2,\ \Delta x = 2 \Rightarrow P(x>8) = 2.77\times10^{-58}$

$n=4,\ \Delta x = 1 \Rightarrow P(x>8) = 1.38\times10^{-58}$

$n=6,\ \Delta x = \frac{2}{3} \Rightarrow P(x>8) = 0.93\times10^{-58}$